Grade Five

In the years prior to grade five, students learned strategies for multiplication and division, developed an understanding of structure of the place value system, and applied understanding of fractions to addition and subtraction with like denominators and to multiplying a whole number times a fraction. Students gained understanding that geometric figures can be analyzed and classified based on their properties and focused on different measurements including angle measures. Students learned to fluently add and subtract whole numbers within 1,000,000 using the standard algorithm (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

WHAT STUDENTS LEARN IN FIFTH GRADE

[Note: Sidebar]

Grade Five Critical Areas of Instruction

In grade five, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume (CCSSO 2010, Grade 5 Introduction).

Students also fluently multiply multi-digit whole numbers using the standard algorithm.

Grade Five Mathematical Content Standards

The Mathematical Content standards emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus:** Instruction is focused on grade level standards.
- **Coherence:** Instruction should be attentive to learning across grades and linking major topics within grades.
- **Rigor:** Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade level examples of focus, coherence, and rigor will be indicated throughout the chapter.

Not all of the content in a given grade is emphasized equally in the standards. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the later demands of college and career readiness.

The following Grade 5 Cluster-Level Emphases chart highlights the content emphases in the standards at the cluster level for this grade. The bulk of instructional time should be given to “Major” clusters and the standards within them. However, standards in the “Supporting” and “Additional” clusters should not be neglected. To do so will result in gaps in students’ learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by utilizing topics in the supporting and additional clusters. Instruction should include problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off a list during isolated units of instruction, but rather content to be developed throughout the school year through rich instructional experiences and presented in a coherent manner (Adapted from the Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

[**Note**: The Emphases chart should be a graphic inserted in the grade level section. The explanation “key” needs to accompany it.]

Grade 5 Cluster-Level Emphases

Operations and Algebraic Thinking

- [a/s]: Write and interpret numerical expressions. (**5.OA.1-2**)
- [a/s]: Analyze patterns and relationships. (**5.OA.3**)

Number and Operations in Base Ten

- [m]: Understand the place value system. (**5.NBT.1-4▲**)
- [m]: Perform operations with multi-digit whole numbers and with decimals to hundredths. (**5.NBT.5-7▲**)

Number and Operations—Fractions

- [m]: Use equivalent fractions as a strategy to add and subtract fractions. (**5.NF.1-2▲**)
- [m]: Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (**5.NF.3-7▲**)

Measurement and Data

- [a/s]: Convert like measurement units within a given measurement system.[1] (**5.MD.1**)
- [a/s]: Represent and interpret data.[2] (**5.MD.2**)
- [m]: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. (**5.MD.3-5▲**)

Geometry

- [a/s]: Graph points on the coordinate plane to solve real-world and mathematical problems. (**5.G.1-2**)
- [a/s]: Classify two-dimensional figures into categories based on their properties. (**5.G.3-4**)

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[3] [m] clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade **Supporting [s]** clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

[1] Work in these standards supports computation with decimals. For example, converting 5 cm to .05 m involves computation with decimals to hundredths.
[2] The standard in this cluster provides an opportunity for solving real-world problems with operations on fractions, connecting directly to both Number and Operations – Fractions clusters.
[3] The ▲symbol will indicate standards in a Major Cluster in the narrative.

(Adapted from Smarter Balanced Assessment Consortia [Smarter Balanced], DRAFT Content Specifications 2012).

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject that makes use of their ability to make sense of mathematics. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grades, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Below are some examples of how the MP standards may be integrated into tasks appropriate for Grade 5 students. (Refer to pages 9–12 in the Overview of the Standards Chapters for a complete description of the MP standards.)

Standards for Mathematical Practice (MP)
Explanations and Examples for Grade Five

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In grade five, students solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. For example, Sonia had $2\frac{1}{3}$ sticks of gum. She promised her brother that she would give him ½ of a stick of gum. How much will she have left after she gives her brother the amount she promised?

	Teachers can encourage students to check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
MP.2 Reason abstractly and quantitatively.	Students recognize that a number represents a specific quantity. They connect quantities to written symbols and create logical representations of problems, consider appropriate units and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Teachers can support student reasoning by asking questions such as, "What do the numbers in the problem represent?"or "What is the relationship of the quantities?" Students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts. For example, students use abstract and quantitative thinking to recognize that $0.5 \times (300 \div 15)$ is $\frac{1}{2}$ of $(300 \div 15)$ without calculating the quotient.
MP.3 Construct viable arguments and critique the reasoning of others.	In fifth grade students may construct arguments using visual models, such as objects and drawings. They explain calculations based upon models, properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. Students use various strategies to solve problems and they defend and justify their work with others. For example, two afterschool clubs are having pizza parties. The teacher will order 3 pizzas for every 5 students in the math club; and 5 pizzas for every 8 students in the student council. If a student is in both groups, decide which party he/she should to attend. How much pizza will each student get at each party? If a student wants attend the party with the most pizza (if divided equally between the students at the party), which party should he/she attend?
MP.4 Model with mathematics	In grade five, students experiment with representing problem situations in multiple ways such as using numbers, mathematical language, drawings, pictures, objects, charts, lists, graphs and equations. Teachers might ask, "How would it help to create a diagram, chart or table?" or "What are some ways to represent the quantities?" Students need opportunities to represent problems in various ways and explain the connections. Fifth graders evaluate their results in the context of the situation and they explain whether results to problems make sense. They evaluate the utility of models they see and draw and can determine which models can be the most useful and efficient to solve problems.
MP.5 Use appropriate tools strategically.	Students consider available tools, including estimation, and decide which tools might help them solve mathematical problems. For instance, students may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions to find a pattern for volume using length of the sides. They use graph paper to accurately create graphs and solve problems or make predictions from real-world data.
MP.6 Attend to	Students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own

precision.	reasoning. Teachers might ask, “How do you know your solution is reasonable?” Students use appropriate terminology when they refer to expressions, fractions, geometric figures, and coordinate grids. Teachers might ask, “What symbols or mathematical notations are important in this problem?” Students are careful to specify units of measure and state the meaning of the symbols they choose. For instance, to determine the volume of a rectangular prism, students record their answers in cubic units.
MP.7 Look for and make use of structure.	Students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation. Teachers might ask, “How do you know if something is a pattern?” or “What do you notice when…?”
MP.8 Look for and express regularity in repeated reasoning.	Fifth graders use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand and use algorithms to extend multi-digit division from one-digit to two-digit divisors and to fluently multiply multi-digit whole numbers. They use various strategies to perform all operations with decimals to hundredths and they explore operations with fractions with visual models and begin to formulate generalizations. Teachers might ask, “Can you explain how this strategy works in other situations?” or “Is this always true, sometimes true or never true?”

(Adapted from Arizona Department of Education [Arizona] 2012 and North Carolina Department of Public Instruction [N. Carolina] 2013)

Standards-based Learning at Grade Five

The following narrative is organized by the domains in the content standards and highlights some necessary foundational skills from previous grades and provides exemplars to explain the content standards, highlight connections to the various Standards for Mathematical Practice (**MP**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (▲) indicates standards in the major clusters (refer to the Grade 5 Cluster-Level Emphases table on page 2.

Domain: Operations and Algebraic Thinking

In preparation for the progression of expressions and equations in the middle grades, students in grade five begin working more formally with expressions.

Operations and Algebraic Thinking	5.OA
Write and interpret numerical expressions.	
1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	

2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
2.1 Express a whole number in the range 0-50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as $2\times2\times2\times3$. CA

Previously in third grade, students began to use the conventional order or operations (e.g., multiplication and division are done before addition and subtraction). In fifth grade, students build on this work to write, interpret and evaluate simple numerical expressions, including those that contain parentheses, brackets, or braces (ordering symbols) (**5.OA.1-2**). Students need opportunities to describe numerical expressions without evaluating them. For example, they express the calculation “add 8 and 7, then multiply by 2” as $(8 + 7) \times 2$. They recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without calculating the sum or product. Students begin to think about numerical expressions in anticipation for their later work with variable expressions (e.g., three times an unknown length is $3 \times L$). (Adapted from Arizona 2012 and Kansas Association of Teachers of Mathematics [KATM] 5^{th} FlipBook 2012).

Students need experiences with multiple expressions to understand when and how to use ordering symbols. Instruction in the order of operations should be carefully sequenced from simple to more complex problems. (Source: Progressions K-5 OA) In grade five, this work should be viewed as exploratory rather than for attaining mastery; for example, expressions should not contain nested grouping symbols, and they should be no more complex than the expressions one finds in an application of the associative or distributive property, such as $(8 + 27) + 2$ or $(6 \times 30) + (6 \times 7)$. (Adapted from The University of Arizona Progressions Documents for the Common Core Math Standards [Progressions], K-5 CC and OA 2011).

Students can begin by using these symbols with whole numbers and then expand the use to decimals and fractions.

Examples:	**Answer:**
$(28 + 16) \div 4$	The answer is 11. Note that if a student gets 32, they may have found $28 + 16 \div 4$.

12 – (2 × 0.4)	The answer is 11.2. Notice that if a student gets 4, they may have found (12 – 2) × 0.4.
(2 + 3) × (1.5 – 0.5)	The answer is 5. Note that if students get 6, they may have found 2 + 3 × 1.5 – 0.5, which yields 6 based on order of operations.
$6-\left(\frac{1}{2}+\frac{1}{3}\right)$	The answer is $5\frac{1}{6}$.

To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently.

Examples:
15 – 7 – 2 = 10 with correct grouping symbols is 15 – (7 – 2) = 10
3 × 125 ÷ 25 + 7 = 22 with correct grouping symbols is 3 × (125 ÷ 25) + 7 = 22
Compare 3 × 2 + 5 and 3 × (2 + 5)
Compare 15 – 6 + 7 and 15 – (6 + 7)

Common Misconceptions.
• Students may believe the order in which a problem with mixed operations is written is the order to solve the problem. The use of the mnemonic phrase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally" to remember the order of operations (**P**arentheses, **E**xponents, **M**ultiplication, **D**ivision, **A**ddition, **S**ubtraction) can also mislead students to always perform multiplication before division and addition before subtraction. To correct this thinking, students need to understand to work from within the inner-most grouping symbols first and that some operations are done before others even without grouping symbols. • Students need lots of experience with writing multiplication in different ways. Multiplication can be indicated with a raised dot, such as "4·5", with a raised cross symbol, such as "4×5", or with parentheses, such as "4(5)" or "(4)(5)." Note that the raised cross symbol is *not the same as* the letter "x," and so care should be taken when writing or typing it. Students need to be exposed to all three notations and should be challenged to understand that all are useful. In instruction, teachers are encouraged to use a notation and stay consistent. Students also need help and practice remembering the convention that we write a rather than 1 · a or 1a, especially in expressions such as a + 3a.

(Adapted from Arizona 2012 and KATM 5th FlipBook 2012).

Operations and Algebraic Thinking 5.OA

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example…*

Understanding patterns is fundamental to algebraic thinking. Students extend their grade four pattern work to include two numerical patterns that can be related and examine these relationships within sequences of ordered pairs and in the graphs in the first quadrant of the coordinate plane (**5.OA.3**). This work prepares students for studying proportional relationships and functions in middle school, and is closely related to graphing points in the coordinate plane (**5.G.1-2).**

Example: Create two sequences of numbers, both starting from 0, but one generated with a "+3" pattern, and the other with a "+ 6" pattern.

a. How are the sequences related to each other?
b. Graph the sequences together as ordered pairs, the first sequence being the *x*-coordinate and the second sequence being the *y*-coordinate.
c. How can you see how the sequences are related together based on the graph?

Solution:

Starting with 0, students create two sequences of numbers:

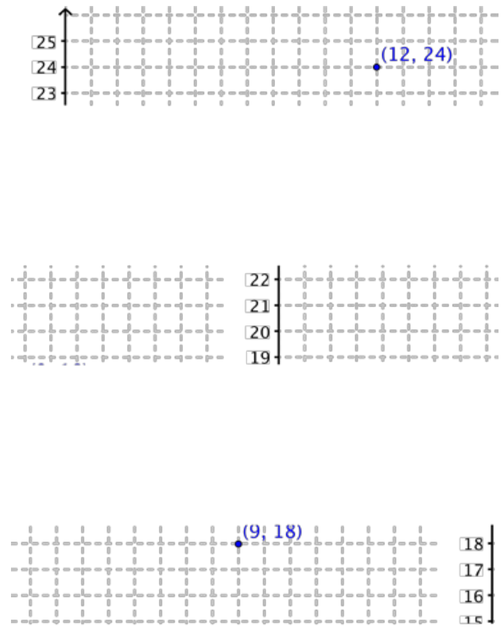
Sequence A:	0,	3,	6,	9,	12,	15,	…
Sequence B:	0,	6,	12,	18,	24,	30,	…

a. Students may notice that each term in sequence B is twice the corresponding term in sequence A. Organizing the sequences in a table as above can help students see the pattern more clearly. Students can explain the relationship between the sequences in several ways, for instance, by using the distributive property:

6 + 6 + 6 = 2× (3 + 3 + 3)

b. The ordered pairs come easily from the table layout: (0,0), (3,6), (6,12), (9,18), etc.

The graph is shown.



c. Students may see that the second coordinate of each point is twice the first, which is natural based on the way the sequences were created. They may also see other features of the graph, such as the "+ 3" pattern moving in the *x* direction and the "+ 6" pattern moving in the *y* direction. (This is fully explored in grades 6-8).

(Adapted from Arizona 2012 and KATM 5th FlipBook 2012)

Common Misconception.

Students reverse the points when plotting them on a coordinate plane. They count up first on the y-axis and then count over on the x-axis

Domain: Number and Operations in Base Ten

In grade five, a critical area of instruction is for students to integrate decimal fractions into the place value system, develop an understanding of operations with decimals to hundredths, and work towards fluency with whole number and decimal operations.

Number and Operations in Base Ten 5.NBT

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros in the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use

whole-number exponents to denote powers of 10.

3. Read, write, and compare decimals to thousandths.

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 247.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4. Use place value understanding to round decimals to any place.

Students extend their understanding of the base-ten system from whole numbers to decimals focusing on the relationship between adjacent place values, how numbers compare, and how numbers round for decimals to thousandths. Before considering the relationship of decimal fractions, students reason that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left (**5.NBT.1▲**).

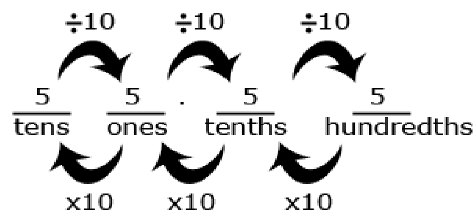
(Adapted from Progressions K-5 NBT 2011).

Example:

Through exploration with base-ten blocks or attaching cubes, students can tangibly explore the relationship between place values. They may be able to name place values, but this is not an indication that they understand the relationship between them. For example, a student may know that the difference between the digits 5 in the number 4554, represent 500 and 50, but the further relationship that 500 = 50 × 10 and 50 = 500 × ($\frac{1}{10}$) needs to be explored and made explicit.

To extend this understanding of place value to their work with decimals, students could use a model of one unit; cut it into 10 equal pieces, shade in, or describe $\frac{1}{10}$ of that model using fractional language ("This is 1 out of 10 equal parts. So it is $\frac{1}{10}$". I can write this using $\frac{1}{10}$ or 0.1"). Students repeat the process by finding $\frac{1}{10}$ of a $\frac{1}{10}$ (e.g., dividing $\frac{1}{10}$ into 10 equal parts to arrive at $\frac{1}{100}$ or 0.01) and explain their reasoning, "0.01 is $\frac{1}{10}$ of $\frac{1}{10}$ thus is $\frac{1}{100}$ of the whole unit." Simple 10 ×10 grids can be very useful for exploring these ideas. Also, since the metric system is a base-10 system of measurement, working with simple metric length measurements and rulers can support this understanding (see standard **5.MD.1**).

In general, students are led to the following pattern: Students recognize that in a multi-digit number:



(Adapted from Arizona 2012 and KATM 5th FlipBook 2012).

Students use place value to understand that multiplying a decimal by 10 results in the decimal point appearing one place to the right (e.g., 10 × 4.2 = 42) since the result is ten times larger than the original number; similarly, multiplying a decimal by 100 results in the decimal point appearing two places to the right since the number is 100 times bigger. Students also make the connection that dividing by 10 results in the decimal point appearing one place to the left (e.g. 4 ÷ 10 = .4) since the number is 10 times smaller (or $\frac{1}{10}$ or the original), and dividing a number by 100 results in the decimal point appearing two places to the left since the number is 100 times smaller (or $\frac{1}{100}$ of the original).

[Note: Sidebar]

Focus, Coherence, and Rigor:

The extension of the place value system from whole numbers to decimals is a major accomplishment involving understanding and skill with base-ten units and fractions (**5.NBT.1▲**). As students understand that in a multi-digit number, a digit in one place represents $\frac{1}{10}$ of what it represents in the place to its left (**5.NBT.1▲**) they also reinforce their understanding of multiplying a quantity by a fraction **(5.NF.4 ▲).** (Adapted from PARCC 2012).

Powers of 10 is a fundamental aspect of the base-ten system. Students extend their understanding of place to explain patterns in the number of zeros of the product when multiplying a number by powers of 10, including the placement of the decimal point. New at grade five is the use of whole number exponents to denote powers of 10 (**5.NBT.2▲**). For example:

Students might write, $36 \times 10 = 36 \times 10^1 = 360$ $36 \times 10 \times 10 = 36 \times 10^2 = 3600$ $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36{,}000$ $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360{,}000$
Students might think and/or say, “I noticed that every time I multiplied by 10 I placed a zero to the end of the number. That makes sense because each digit’s value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left. When I multiplied 36 by 10, the 30 became 300. The 6 became 60 (or the 36 became 360).

(Adapted from Arizona 2012)

[Note: Sidebar]

Focus, Coherence, and Rigor:

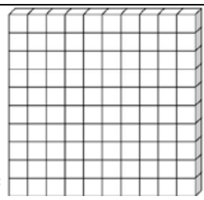
Students can use their understanding of the structure of whole numbers to generalize this understanding to decimals (**MP.7**) and explain the relationship between the numerals (**MP.6**) (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

Students build on understandings from fourth grade to read, write, and compare decimals to thousandths **(5.NBT.3▲).** They connect this work with prior understanding of decimal notations for fractions and addition of fractions with denominators of 10 and 100. Students use concrete models or drawings and number lines to extend this understanding to decimals to the thousandths. Models may include base-ten blocks, place value charts, grids, pictures, drawings, manipulatives and technology. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800). For example:

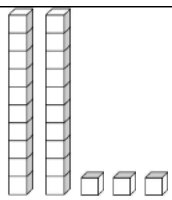
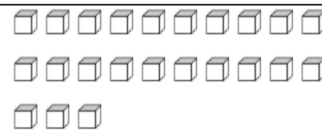
Equivalent forms of 0.72:	
$\frac{72}{100}$	$\frac{70}{100} + \frac{2}{100}$
$\frac{7}{10} + \frac{2}{100}$	0.720
$7 \times \frac{1}{10} + 2 \times \frac{1}{100}$	$7 \times \frac{1}{10} + 2 \times \frac{1}{100} + 0 \times \frac{1}{1000}$

$0.70 + 0.02$	$\frac{720}{1000}$

Base-10 blocks can be a powerful tool for seeing these representations. For instance, if a "flat" is used to represent 1 (the whole or unit), then a "stick" represents 1/10, and a small "cube" represents 1/100. Students can be challenged to make sense of a number like 0.23 as being represented by both 2/10 + 3/100 and 23/100:

If  represents 1, then  represents $\frac{1}{10}$ and  represents $\frac{1}{100}$.

"Explain why the following both represent the number 0.23"

"Well, I see that the 20 hundredths in the picture on the right can be grouped into 2 sets of 10 hundredths. That means these 2 groups represent 2 tenths, or $\frac{2}{10}$. There are 3 hundredths left, so altogether there are $\frac{2}{10} + \frac{3}{100}$."

Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.

Example:
Comparing 0.207 to 0.26, a student might think, "Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, I know that 0.207 is 207 thousandths (and may write $\frac{207}{1000}$) and 0.26 is 26 hundredths (and may write $\frac{26}{100}$) but I can also think of it

as 260 thousandths ($\frac{260}{1000}$). So, 260 thousandths is more than 207 thousandths.

For students who are not able to read, write, and represent multi-digit numbers, working with decimals will be challenging. Money is a good medium to provide meaning for decimals (e.g., dimes can represent tenths, pennies represent hundredths, and a penny circle with a $\frac{1}{10}$ sliver in it can represent thousandths), as well as base-10 blocks like those used above.

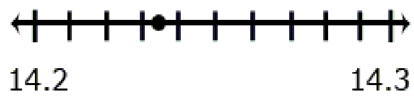
Reading decimals can confuse some students because numbers to the left of the decimal are read based on the place value of the digit farthest to the left of the decimal (e.g., 462 is read as "four hundred sixty two"); however, decimals are read as whole numbers with the hundreds, tens, and ones said and then the place value of the digit farthest to the right of the decimal (e.g., .46 is read as 46 hundredths). Decimals are read as if they are fractions: You read the number as the numerator and then say the denominator.

Common Misconception.
Some students relate comparing decimals with the idea "the longer the number the greater the number." With whole numbers, a five-digit number is always greater that a one-, two-, three-, or four-digit number. However, when comparing decimals, a number with one decimal place may be greater than a number with two or three decimal places

Students use place value understanding to round decimals to any place (**5.NBT.4▲**). When rounding a decimal to a given place, students may identify two possible answers and use their understanding of place value to compare the given number to the possible answers.

Example: Round 14.235 to the nearest tenth.
Students can read 14.235 as "14 and 235 thousandths." Since they are rounding to the nearest tenth, they are most likely rounding to either 14.2 or 14.3, that is, "14 and 200 thousandths" and "14 and 300 thousandths" (14.200 and 14.300). Students then see that they can disregard for the moment the 14 and

just focus on rounding 235 (thousandths) to the nearest hundred. In that case, since 235 would round down to 200, we'd get 14.200.



Students can use benchmark numbers (e.g., 0, 0.5, 1, and 1.5) to support similar work.

Number and Operations in Base Ten 5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties or operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

In grades three and four, students used various strategies to multiply. In grade five students fluently multiply multi-digit whole numbers using the standard algorithm (**5.NBT.5▲**). Generally the standards distinguish strategies from algorithms. In particular, the "standard algorithm" refers here to multiplying numbers digit-by-digit and recording the products piece-by-piece. Note that the *method of recording* the algorithm is not the same as the *algorithm* itself, in the sense that the "partial products" method, which lists every single digit-by-digit product separately, is a completely valid recording method for the "standard algorithm." Ultimately, the standards call for *understanding* the standard algorithm in terms of place value, and this should be the most important goal for instruction.

[Note: Sidebar]

FLUENCY

In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., "fluently" multiply multi-digit whole numbers using the standard algorithm (**5.NBT.5▲**). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary.

The word “fluent” is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.

In previous grades, students built a conceptual understanding of multiplication with whole numbers as they applied multiple strategies to compute and solve problems. Students can continue to use different strategies and methods from previous years as long as they are efficient, but they must also understand and be able to use the standard algorithm.

Example: Find the product 123 × 34

When students apply the standard algorithm, they decompose 34 into 30 + 4. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products. The ways in which students are taught to record this method may vary, but all should emphasize the place-value nature of the algorithm. For example, one might write

$$\begin{array}{rl} 123 & \\ \times\ 34 & \\ \hline 492 & \leftarrow \text{this is the product of 4 and 123} \\ 3690 & \leftarrow \text{this is the product of 30 and 123} \\ \hline 4182 & \leftarrow \text{this is the sum of the two partial products} \end{array}$$

Note that a further decomposition of 123 into 100 + 20 + 3 and recording of the partial products would also be acceptable.

(Adapted from Arizona 2012).

In grade five students extend division to include quotients of whole numbers with up to four-digit dividends and two-digit divisors using various strategies, and they illustrate and explain calculations by using equations, rectangular arrays, and/or area models. (**5.NBT.6▲**). When the two-digit divisor is a “familiar” number, students might use various strategies based on place value understanding.

Example 1: Find the quotient 2682 ÷ 25

- Using expanded notation: 2682 ÷ 25 = (2000 + 600 + 80 + 2) ÷ 25
- Using an understanding of the relationship between 100 and 25, a student might think:
  - I know that 100 divided by 25 is 4 so 200 divided by 25 is 8 and 2000 divided by 25 is 80.
  - 600 divided by 25 has to be 24.
  - Since 3 x 25 is 75, I know that 80 divided by 25 is 3 with a reminder of 5. (Note that a student might divide into 82 and not 80)
  - I can't divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
  - 80 + 24 + 3 = 107. So, the answer is 107 with a remainder of 7.
- Using an equation that relates division to multiplication, 25 × n = 2682, a student might estimate the answer to be slightly larger than 100 by recognizing that 25 × 100 = 2500.

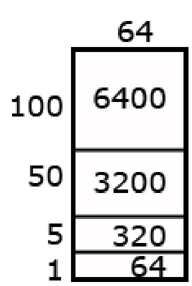
To help students understand the use of place value when dividing with two digit divisors, students can begin with simpler examples, such as dividing150 by 30. Clearly the answer is 5 since this is 15 tens divided by 3 tens. However, when dividing 1500 by 30, students need to think of this as 150 tens divided by 3 tens, which is 50. This illustrates why when using the division algorithm the 5 would go in the tens place of the quotient.

When the divisor is less familiar, students can use strategies based on area such as shown in the following example.

Example 2: Find the quotient 9984 ÷ 64

An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.

Area model:



So the quotient is 100 + 50 + 5 + 1 =156.

Recording:

64	9984	
	−6400	(100 × 64)
	3584	
	−3200	(50 × 64)
	384	
	−320	(5 × 64)
	64	
	−64	(1 × 64)
	0	

(Adapted from Arizona 2012)

The extension from one-digit divisors to two-digit divisors requires care (**5.NBT.6▲**). This is a major milestone along the way to reaching fluency with the standard algorithm in grade six. Division strategies in grade five extend the grade four methods to 2-digit divisors. Students continue to break the dividend into base-ten units and find the quotient place by place, starting from the highest place. They illustrate and explain their calculations using equations, rectangular arrays, and/or area models. Estimating the quotients is a difficult new aspect of dividing by a 2-digit number. Even if students round appropriately, the resulting estimate may need to be adjusted up or down. Students may write any needed new group from multiplying within the division or add it in mentally or write the multiplication out to the side, if necessary.

[Note: Sidebar]

Focus, Coherence, and Rigor: When students break divisors and dividends into sums of multiples of base-ten units (**5.NBT.6▲**), they also develop important mathematical practices such as how to see and make use of structure (**MP.7**) and attend to precision (**MP.6**). (PARCC 2012).

In grade five students build on work with comparing decimals in fourth grade and begin to add, subtract, multiply, and divide decimals to hundredths (**5.NBT.7▲**). Students focus on reasoning about operations with decimals using concrete models, drawings, various strategies, and explanations. They extend the models and written models they developed for whole numbers in grades one through four to decimal values.

Students might estimate answers based on their understanding of operations and the value of the numbers. **(MP.7, MP.8)**

Examples: Estimate
3.6 + 1.7. A student can make good use of rounding to estimate that since 3.6 rounds up to 4 and 1.7 rounds up to 2, the answer should be close to 4 + 2 = 6.
5.4 − 0.8. Students can again round and argue that since 5.4 rounds down to 5 and 0.8 rounds up to 1, the answer should be close to 5 – 1 = 4.
6 × 2.4. A student might estimate an answer between 12 and 18 since 6 × 2 is 12 and 6 × 3 is 18.

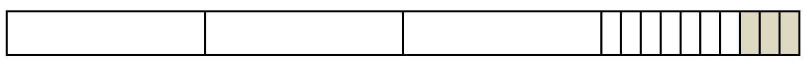
Students must understand and be able to explain that when adding decimals they add tenths to tenths and hundredths to hundredths. When students add in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. Students reinforce their understanding of adding decimals by connecting to prior understanding of adding fractions with denominators of 10 and 100 from fourth grade. Students understand that when adding and subtracting a whole number the decimal point is at the end of the whole number.

Students use various models to support their understanding of decimal operations.

Example 1: (Model for decimal subtraction)

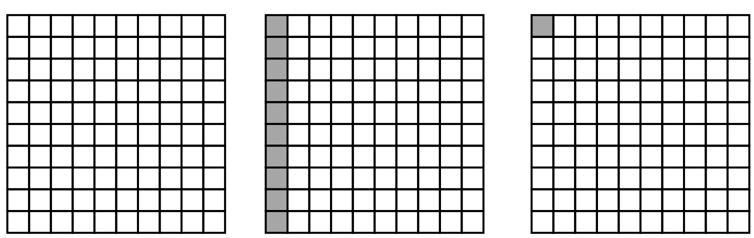
Find $4 - 0.3$. Explain how you found your solution.

"Since I'm subtracting 3 tenths from 4 wholes, it would help to divide one of the wholes into tenths. The other 3 wholes don't need to be divided up. I can see there are 3 wholes and 7 tenths leftover, or 3.7."



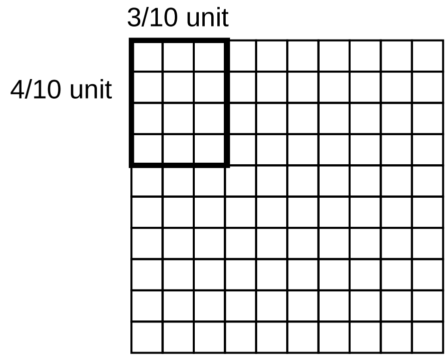
Example 2: Use an area model to demonstrate that $\frac{1}{10}$ of $\frac{1}{10}$ is $\frac{1}{100}$.

"If I use my 10×10 grid and set the whole grid to equal 1 square unit, then I can see that when each length of the grid is divided into ten equal parts, each small square must be representing a $\frac{1}{10}\times\frac{1}{10}$ square. But there are 100 of these small squares in the whole, so each little square must have area $\frac{1}{100}$ square units."



Example 3: Use an area model to demonstrate that $\frac{3}{10}\times\frac{4}{10}=\frac{12}{100}$.

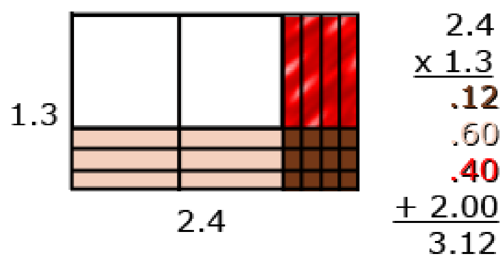
"Just like in the previous problem, I use my 10×10 grid to represent 1 whole, with dimensions 1 unit by 1 unit. If I break up each side length into ten equal parts, then I can create a smaller rectangle of dimensions 3 tenths of a unit by 4 tenths of a unit. It looks something like this:



I know from before that each of the small squares is $\frac{1}{100}$ of a square unit, and I can see there are 3 × 4 = 12 of these small squares in the rectangle I outlined. This shows the answer is $\frac{12}{100}$." (See also **5.NF.4)**

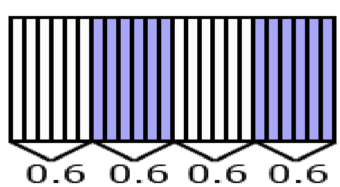
Example 4: Use an area model to show that 2.4 × 1.3 = 3.12.

"I drew a picture that shows a rectangle of lengths 1.3 units and 2.4 units. I know how to break up and keep track of the smaller units like tenths and hundredths. The partial products appear in my picture a lot like the previous problem."



Example 5: (Partitive or "fair-share" division model applied to decimals.) Find 2.4 ÷ 4 and justify your answer.

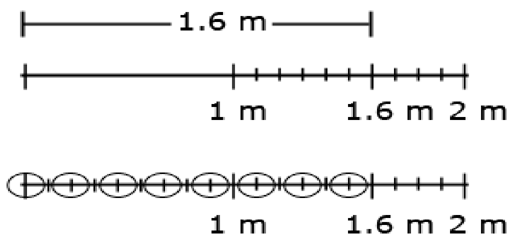
"My partner and I decided to think of this as fair-share division. We drew 2 wholes and 4 tenths, and decided to break the wholes into tenths as well, since it would be easier to share them. When we tried to divide the total number of tenths into four equal parts, we got 0.6 in each part."



Example 6: (Quotitive or "measurement" division model applied to decimals.)

Solve the following problem: "Joe has 1.6 meters of rope. He needs to cut pieces of rope that are 0.2 meters long. How many pieces can he cut?

"We decided to draw a number line segment 2 units long that would represent Joe's 1.6 meters of rope, 1 whole meter and 6 tenths of a meter. Since we need to count smaller ropes that are 0.2 meters in length, we also decided to divide the 1 whole into tenths as well. Then it wasn't too hard to count that there are 8 pieces of 0.2-meter long rope in his 1.6-meter rope."



(Adapted from Arizona 2012 and KATM 5th FlipBook 2012)

Domain: Number and Operations-Fractions

Student proficiency with fractions is essential to success in algebra at later grades. In grade five a critical area of instruction is developing fluency with addition and subtraction of fractions, including adding and subtracting fractions with unlike denominators. Students also build an understanding of multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

Number and Operations—Fractions	5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.*
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases with unlike denominators, e.g., by using visual fraction models or equations to represent the problems. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

In grade four students calculated sums of fractions with different denominators, where one denominator is a divisor of the other, so that only one fraction has to be changed. In grade five students extend work with fractions to add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions with like denominators **(5.NF.1▲).** (Adapted from Progressions 3-5 NF 2011)

Students find a common denominator by finding the product of both denominators. For $\frac{1}{3}+\frac{1}{6}$, a common denominator is 18, which is the product of 3 and 6. This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm. Student should first solve problems that require changing one of the fractions (as in grade four) and progress to changing both fractions. Students understand that multiplying the denominators will always give a common denominator but may not result in the smallest denominator; however, it is not necessary to find a least common denominator to calculate sums and differences of fractions.

To add or subtract fractions with unlike denominators, students need an understanding of how to create equivalent fractions with the same denominators before adding or subtracting, a concept learned in grade 4. In general they understand that for any whole numbers a, b , and n , $\frac{a}{b}=\frac{n \times a}{n \times b}$ (given that n and b are non-zero).

Examples:
$\frac{2}{5}+\frac{7}{8}=\frac{2}{5}\cdot\frac{8}{8}+\frac{7}{8}\cdot\frac{5}{5}=\frac{16}{40}+\frac{35}{40}=\frac{51}{40}$
$3\frac{1}{4}-\frac{1}{6}=3\frac{6}{24}-\frac{4}{24}=3\frac{2}{24}\ \text{or} 3\frac{1}{12}$

(Adapted from Progressions 3-5 NF 2011)

Students make sense of fractional quantities when solving word problems involving addition and subtraction of fractions referring to the same whole using a variety of strategies **(5.NF.2▲)**.

Example: Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the

other needed 2/3 of a cup of sugar. How much sugar did he need to make both recipes?

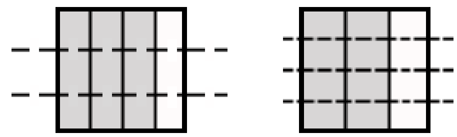
Solutions:

Mental Estimation (MP.2): A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups, since both fractions are larger than 1/2, while at the same time both fractions are less than 1.

Area Model to show equivalence (MP.5): A student may choose to represent each partial cup of sugar using an area model, find equivalent fractions, and then add:

I see that 3/4 of a cup of sugar is equivalent to 9/12 of a cup, while 2/3 of a cup is equivalent to 8/12 of a cup. Altogether, I have 17/12 of a cup. This is more than one cup since

$\frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}$.



$\frac{3}{4}$ cup of sugar

$\frac{2}{3}$ cup of sugar

(Adapted from Arizona 2012)

[Note: Sidebar]

Focus, Coherence, and Rigor:

When students meet standard **(5.NF.2▲),** they bring together the threads of fraction equivalence (learned in grades three through five) and addition and subtraction (learned in kindergarten through grade four) to fully extend addition and subtraction to fractions**.** (Adapted from PARCC 2012).

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

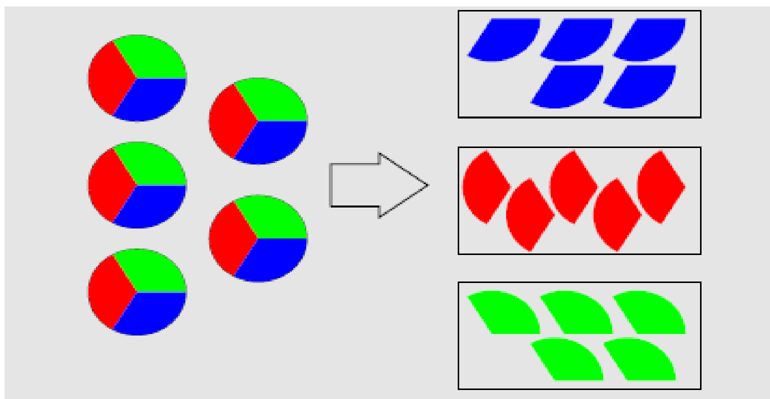
   a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3)× 4=8/3, and create a story context for this equation. Do the same with (2/3)× (4/5) = 8/15. (In general, (a/b)× (c/d) = ac/bd.)*

   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

In grade five students connect fractions with division, understanding that 5 ÷ 3 = 5/3 or, more generally, *a/b* = *a* ÷ *b* for whole numbers *a* and *b*, with *b* not equal to zero **(5.NF.3▲).** Students can explain this by working with their understanding of division as equal sharing.

For example: Sharing 5 objects equally among three shares, showing that

$5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$



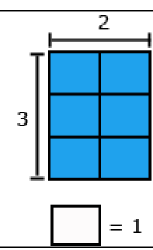
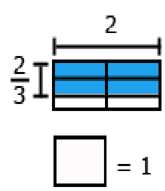
If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute 1/3 of itself to each share. Thus each share consists if 5 pieces, each of which is 1/3 of an object, so each share is 5× 1/3 = 5/3 of an object. (Progressions 3-5 NF 2012)

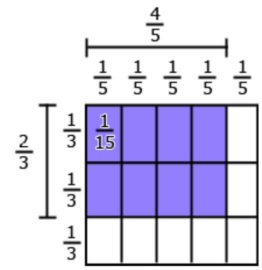
Students solve related word problems and demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$ as "three-fifths" and, after experiences with sharing problems, students generalize that dividing 3 into 5 equal parts (3 ÷ 5 also written as $\frac{3}{5}$) results in the fraction $\frac{3}{5}$ (3 of 5 equal parts).

Students apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction **(5.NF.4▲).** Students multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately and solve problems in both contextual and non-contextual situations. Students reason about how to multiply fractions using fraction strips and number line diagrams. Using an understanding of multiplication by a fraction, students develop an understanding of a general formula for the product of two fractions, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

Examples:
When students multiply fractions such as in the problem $\frac{3}{5} \times 35$, they can think of the operation in more than one way: • As $3 \times (35 \div 5)$, or $3 \times \frac{35}{5}$. (This is equivalent to $3 \times \left(\frac{1}{5} \times 35\right)$ and expresses the idea in standard **5.NF.4.b▲)**. • As (3 × 35) ÷ 5, or 105 ÷ 5. (This is equivalent to $\frac{105}{5}$.)
Students may be challenged to write a story problem for this operation. "Mark's mother said he could have $\frac{3}{5}$ of the peanuts she bought for him and his younger brother to share. If she bought a bag of 35 peanuts, how many peanuts does Mark receive?"

Building on previous understandings of multiplication, students find the area of a rectangle with fractional side lengths and represent fraction products as areas.

Examples of the reasoning called for in standard 5.NF.4.b▲.	
Students have previously worked with examples of finding products as finding areas. In general, the factors in a multiplication problem represent the lengths of a rectangle and the product represents the area.	"This rectangle shows me that 2 × 3 = 6 by counting the number of square units and the side lengths of the rectangle." 
When students move to examples like $2 \times \frac{2}{3}$, they recognize that one side of the rectangle is less than a unit length (in this case; some might have mixed number side lengths). The idea of the picture is the same, but finding the area of the rectangle can be a	"I made a rectangle of sides 2 units and 2/3 of a unit. I can see that the two unit squares in the pictures are each divided into three equal parts (representing 1/3), with two shaded in each unit square (4 total). That means that 

little more challenging and requires reasoning about unit areas and how many parts unit areas are being divided into.	altogether the area of the shaded rectangle is 4/3 square units.”
Finally, when students move to examples like $\frac{2}{3} \times \frac{4}{5}$, they see that the division of the side lengths into fractional parts creates a division of the unit area into fractional parts as well. Students will discover that the fractional parts of the unit area are related to the denominators of the original fractions. Here, a 1×1 square is divided into thirds in one direction and fifths in another. This results in the unit square itself being divided into fifteenths. This reasoning shows why $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$.	“I created a unit square, divided it into fifths in one direction and thirds in the other. This allows me to shade a rectangle of dimensions 2/3 and 4/5. I  noticed that 15 of the new little rectangles make up the entire unit square, so they must be fifteenths (1/15). Altogether, I had 2×4 of those fifteenths. So my answer is 8/15.”

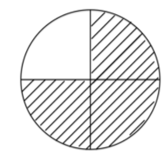
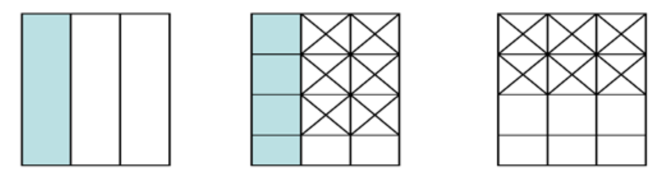
(Adapted from Arizona 2012)

[Note: Sidebar]

Focus, Coherence, and Rigor: When students meet standard (**5.NF.4▲**), they fully extend multiplication to fractions, making division of fractions in grade six (**6.NS.1**) a near target.

Following is a sample classroom activity that connects the Standards for Mathematical Content and the Standards for Mathematical Practice.

Connecting to the Standards for Mathematical Practice—Grade 5

Standard(s) Addressed	***Example(s) and Explanations***
5.NF.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (*a*/*b*)×*q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3)×(4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be by multiplying the side lengths. c. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	**Task:** The following sequence of problems can be presented to students with tools provided (colored counters, rectangular and circular fraction pieces, fraction strips or rods, graph paper, etc.). Students should be encouraged to use their tools to solve the problem before presenting algorithms for the computations involved. 1. There are 18 marbles in a box. Two-thirds of the marbles are red. How many red marbles are there? *Solution*: By seeing the 18 marbles as 3 sets of 6, we see that the solution is that 2×6 = 12 marbles are red. Notice that we found thirds of 18 first $\left(\frac{1}{3}\times 18 = 6\right)$ and then decided we wanted two-thirds. Several examples like this can used to show that $\left(a/b\right)\times q = a\times\left(q/b\right)$. 2. Roberto had $\frac{3}{4}$ of a pizza left. He gave $\frac{1}{3}$ of the leftover pizza to his kid sister. How much of the whole pizza did his sister get? *Solution*: Using the same reasoning above, and using pictures to support the reasoning, we can see that one-third of three-fourths is one-fourth, so that Roberto's sister got $\frac{1}{4}$ of a whole pizza.  3. Mr. Jones was mowing his lawn and had $\frac{2}{3}$ of the lawn left to cut before he had to answer a phone call. After the call, he finished $\frac{3}{4}$ of what he had left. How much of the lawn did Mr. Jones cut after the phone call? *Solution*: Here, we add the complication of finding fourths of thirds, which gives twelfths. In total, Mr. Jones has cut six of those twelfths, so the answer is $\frac{6}{12}=\frac{1}{2}$ of the lawn. This can be illustrated with a rectangular fraction picture as shown. The lawn is first divided into thirds, one of which is shaded. Then the lawn is divided into fourths, and we notice that each of the small rectangular pieces represents $\frac{1}{12}$ of the entire lawn. Six of those are outlined in the picture.  Originally he mowed 1/3 of yard. He then mowed ¾ of what was left. How much of the total did he mow? Rearrange the 6 pieces and it is ½ of the area. **Classroom Connections**

	By building up students' understanding of fraction operations with different fraction models, a foundation can be laid for the algorithms to come. For example, eventually, students can attempt to justify the algorithm for multiplying fractions, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ by understanding that first $\frac{c}{d}$ can be divided into b equal parts; then, a of those parts are taken. In total, ac total parts of size $\frac{1}{bd}$ are taken. **Connecting to the Standards for Mathematical Practice** (MP.1) Students can be challenged to make sense out of each problem situation and to use their prior knowledge of fractions to try to model the situation and persevere in solving each problem. (MP.4) Students are using fractional representations to model simple real-world situations. The real-world problems drive the mathematical concepts, as opposed to the opposite approach of learning algorithms and later applying them. (MP.5) Students should have some familiarity with various fraction models and have the opportunity here to use them to solve actual problems and develop a conceptual understanding of fraction operations.

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b* = (*n*× *a*)/(*n*× *b*) to the effect of multiplying *a/b* by 1.

6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

In preparation for grade six work in ratios and proportional reasoning, students interpret multiplication as scaling (resizing) (**5.NF.5▲**) by examining how numbers change as they multiply by fractions. Students should have ample opportunities to examine the following cases: a) that when multiplying by a fraction greater than 1, the number increases, and b) that when multiplying by a fraction less the one, the number decreases. This is a new interpretation of multiplication and one that needs extensive discussion and explanation by students.

Examples:

"I know $\frac{3}{4} \times 7$ is less than 7, because I make 4 equal shares from 7 but I only take 3 of them (¾ is a fractional part less than one). If I'm taking a fractional part of 7 that is less than 1, the answer should be less than 7."

"I know that $2\frac{2}{3} \times 8$ should be more than 8, because 2 groups of 8 is 16 and $2\frac{2}{3} > 2.$ Also, I know the answer should be less than 24 = 3 × 8, since $2\frac{2}{3} < 3.$"

"I can show by equivalent fractions that $\frac{3}{4} = \frac{3 \times 5}{4 \times 5}$. But I also see that $\frac{5}{5} = 1$, so the result should still be equal to $\frac{3}{4}$."

Students apply their understanding of multiplication of fractions and mixed numbers to solve real-world problems using visual models or equations **(5.NF.6▲)**.

Number and Operations—Fractions	5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
   a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*
   b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × 1/5 = 4.*
   c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

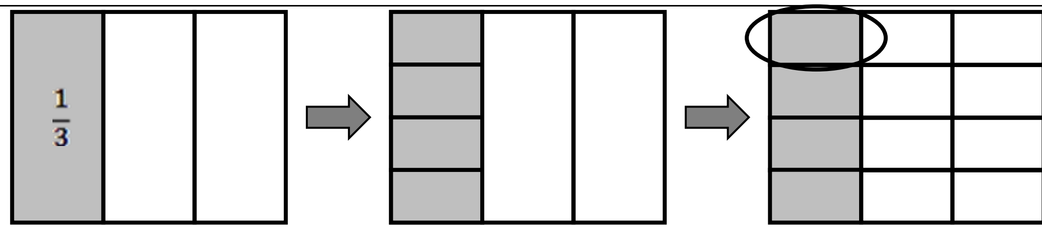
Students apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions **(5.NF.7▲)**, a new concept at fifth grade. Students will extend their learning about division of fractions in simpler cases here in grade five to the general case in grade six (division of a fraction by a fractions is not a requirement at this grade). Students use visual fractions models to show the quotient and solve related real-world problems.

Examples of the reasoning called for in standard (5.NF.7▲)

Partitive Division (Fair-Share Division) for dividing a unit fraction by a whole number:

Four students sitting at a table were given 1/3 of a pan of brownies to share equally. What fraction of a pan of brownies will each student get if they share the pan of brownies equally?

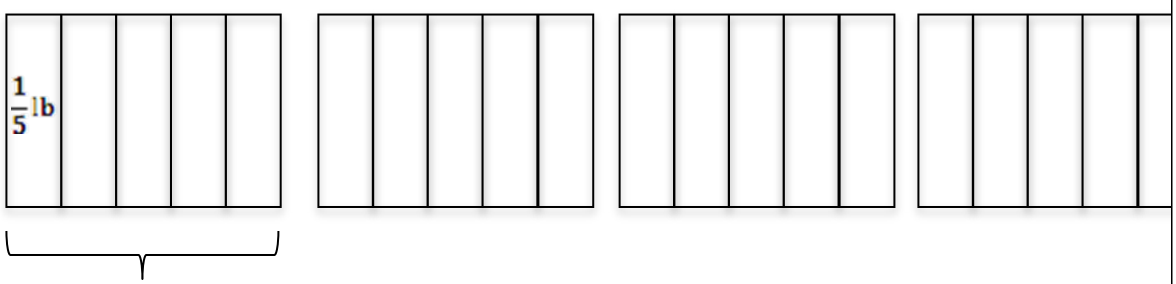
Solution: The diagram shows the 1/3 of a pan of brownies divided into four equal shares. When replicated to fill out the entire pan, it becomes clear that each piece is 1/12 of an entire pan. (Indeed, if the 1/3-sized pieces are each in turn divided into 4 equal pieces, this makes a total of 12 equal pieces of the original whole.)



Students express their problem with an equation and relate it to their visual model: 1/3 ÷ 4 = 1/12, which is the same as ¼ x 1/3. (MP.2, MP.4)

Quotitive Division (Measurement Division) for dividing a whole number by a unit fraction: Angelo has 4 pounds of peanuts. He wants to give each of his friends 1/5 of a pound. How many friends can receive 1/5 lb. of peanuts?

Solution: The question is asking how many groups of sized 1/5 lb. are found in 4 (whole) pounds? This leads us to draw 4 wholes, divide each of them into 1/5-lb. pieces, and count up how many of these pieces are shown.



1 lb of peanuts

We see that there are 20 (twenty) 1/5-lb sized portions in the original 4 pounds. (Alternatively, a student may reason that since there are 5 (1/5-lb) size portions in each individual pound, and so there are 5 × 4 = 20 total. This reasoning lends itself to proportional reasoning in grades 6 and 7.)

(Adapted from Arizona 2012 and KATM 5th FlipBook 2012)

Domain: Measurement and Data

In grade five a critical area of instruction is to develop an understanding of volume. Students recognize volume as an attribute of three-dimensional space. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume.

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.
1. Convert among different-sized standard units within a given measurement system (e.g., convert 5 cm to 0.05 cm), and use these conversions in solving multi-step, real world problems.

In grade five students build on prior knowledge from grade four to express measurements in larger or smaller units within a measurement system (**5.MD.1**). This is an opportunity to reinforce notions of place value for whole numbers and decimals and connections between fractions and decimals (e.g., $2\frac{1}{2}$ meters can be expressed as 2.5 meters or 250 centimeters). Students use these conversions in solving multi-step, real-world problems. (Adapted from Progressions K-5 MD, measurement part 2012)

[Note: Sidebar]

Focus, Coherence, and Rigor:
Students' work with conversions within the metric system **(5.MD.1)** provides opportunities for practical applications of place value understanding and supports major work at the grade in the cluster "Understand the place value system" **(5.NBT.1▲)**.

Measurement and Data	5.MD

Represent and interpret data.
2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Students continue to extend their understanding of representing data, in this case including fractional quantities in real-world situations of data.

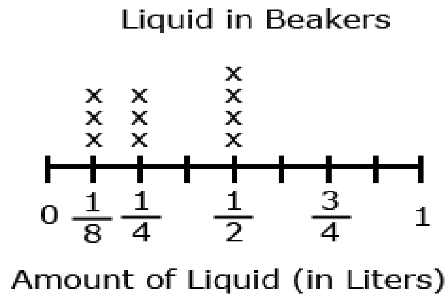
Example: Ten beakers, measured in liters, are filled with a liquid. The numbers of each beaker filled with a certain amount of liquid are represented in the line plot below. If the liquid is redistributed equally, how much liquid will each beaker have?

Students apply their understanding of operations with fractions and use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers. The graph shows that there is a total amount of liquid (in liters) of:

$3 \times \frac{1}{8} + 3 \times \frac{1}{4} + 4 \times \frac{1}{2} = \frac{3}{8} + \frac{3}{4} + \frac{4}{2} = \frac{25}{8}$.

If this 25/8 L of liquid is distributed among the 10 beakers, then there must be $\frac{25}{8} \div 10$ L in each beaker.

Since $\frac{25}{8} = 3\frac{1}{8} = 3.125$, we see that each beaker would contain 0.3125 L of liquid.



(Adapted from Arizona 2010 and KATM 5th FlipBook 2012)

[Note: Sidebar]

Focus, Coherence, and Rigor:

As students solve real-world problems using operations on fractions based on information presented in line plots they reinforce and support major work at the grade in the clusters in the domain **(5.NF),** “Number and Operations—Fractions”.

Measurement and Data 5.MD

Geometric measurement: understand concepts of volume and relate volume to multiplication and addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
    a. A cube with die length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
    b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

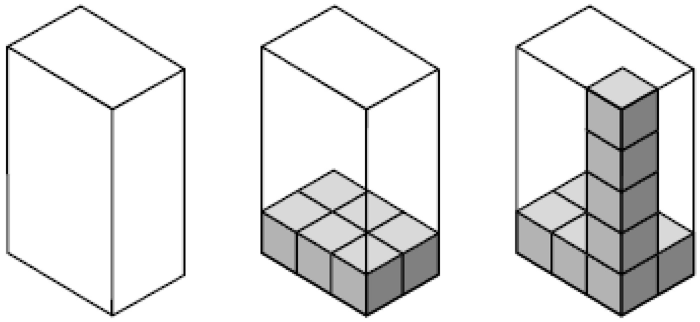
5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
    a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
    b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
    c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding volumes of the non-overlapping parts,

applying this technique to solve real world problems.

Students develop an understanding of volume and relate volume to multiplication and addition. Volume introduces three-space dimension, a significant challenge to some students' spatial structuring and also a complexity in the nature of the materials measured. (**5.MD.3▲**) Solid units are "packed," such as cubes in a three-dimensional array, whereas a liquid "fills" three-dimensional space, taking the shape of the container. "Packing" volume is more difficult than area concepts in early grades (e.g., iterating a unit to measure length and measuring area by tiling). Helping students think of volume as the number of cubes in *n* layers with a given area can be simpler than thinking of all three dimensions. (Adapted from PARCC 2012 and Progressions K-5 MD, measurement part 2012).

Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a "unit cube" (**5.MD.3▲**). They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build (**5.MD.4▲).** Students can also build up a rectangular prism with cubes to see the volume; it is easier to see the cubes in this method.

In grade three students measured and estimated liquid volume and worked with area measurement. At grade five, the concept of volume can be extended from area by relating earlier work covering an area to the bottom of cube with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer. For example:



one layer

five layers fill the box

- (3×2) represented by first layer
- $(3 \times 2) \times 5$ represented by number of 3×2 layers
- $(3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) = 6 + 6 + 6 + 6 + 6 = 30$ (6 representing the size/area of one layer)

Students can apply these ideas by filling containers with cubic units (wooden cubes) to find the volume or building up without the containers. Students may also use drawings or interactive computer software to simulate the same filling process. It is helpful for students use with concrete manipulatives before moving to pictorial representations.

Students measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units **(5.MD.5▲)**.

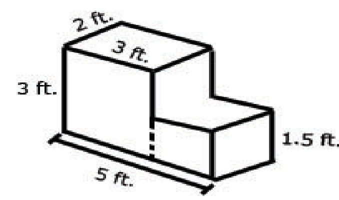
Examples:

Students can be given 24 “unit” cubes and asked to make as many rectangular prisms as possible. Students build the prisms and record the dimensions as they build. It is important to note that there is a constant volume in this activity, and that the product of the length, width, and height of each prism will always be 24.

Length	Width	Height

1	2	12
2	2	6
4	2	3
8	3	1

Students can be asked to determine the volume of concrete needed to build the steps shown in the diagram. **(5.MD.5.c.)**



(Adapted from Arizona 2010)

Focus, Coherence, and Rigor:

When students show that the volume of a right rectangular prism is the same as would be found by multiplying the side lengths (**5.MD.5▲**), they also develop important mathematical practices such as look for and express regularity in repeated reasoning (**MP.8**). They attend to precision (**MP.6**) as they use correct length or volume units, and they use appropriate tools strategically (**MP.5**) as they understand or make drawings to show these units.

Domain: Geometry

Geometry	5.G

Graph points on the coordinate plane to solve real world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pairs of numbers, called its coordinates. Understand that the first number indicates how far travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

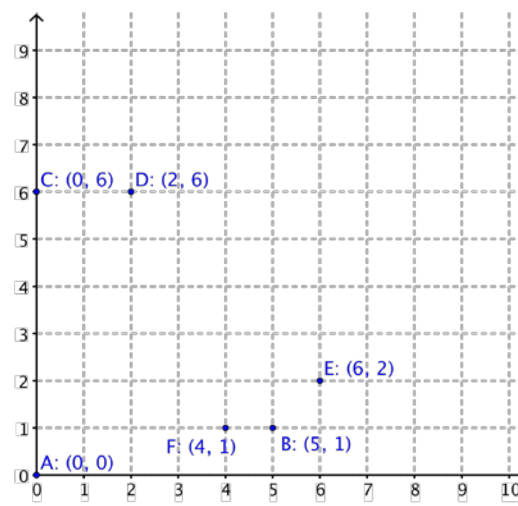
In grade five students build on their previous work with number lines to use two perpendicular number lines to define a coordinate system (**5.G.1**). Students gain an understanding of the structure of the coordinate system. They learn the two axes make it possible to locate points on a coordinate plane and the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). This is the first time students work with coordinate planes, and at grade five this work is limited to the first quadrant.

Students need opportunities to create a coordinate grid, connect ordered pairs of coordinates to points on the grid, and describe how to get to the location (e.g., initially, an ordered pair (2, 3) could be described as a distance “2 from the origin along the x-axis and then 3 units up from the y-axis” or “right 2 and up 3”). For example:

Students might use a classroom size coordinate system to physically locate the coordinate points. For example, to locate the ordered pair (5, 3) students start at the origin point (0,0), then walk 5 units along the *x*-axis to find the first number in the pair (5), and then walk up 3 units for the second number in the pair (3). They continue this process to locate all the points in the following chart. Students recognize that ordered pairs name points in the plane.

Students graph and label the points below in a coordinate system.

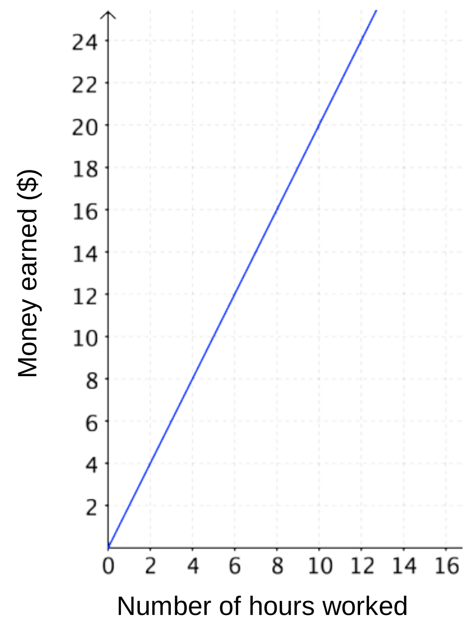
- A (0, 0)
- B (5, 1)
- C (0, 6)
- D (2, 6)
- E (6, 2)
- F (4, 1)



Students represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane (**5.G.2**).

Example: Use the following graph to determine how much allowance Jack makes after doing chores for exactly 10 hours.

Solution: "I can see that when I look up from the *x*-coordinate on the horizontl axis, the *y*-coordinate that matches up to it is 20. So Jack makes $20 if he does 10 hours of chores.



(Adapted from Arizona 2010 and KATM 5th FlipBook 2012)

Focus, Coherence, and Rigor:
Students can connect their work with numerical patterns (**5.OA.3**) to form ordered pairs and to graph these ordered pairs in the coordinate plane (**5.G.1-2**) and then use this model to make sense of and explain the relationships within the numerical patterns they generate. This work can help prepare students for future work with functions and proportional relations in the middle grades. (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

Common misconception: Students may think the order in plotting a coordinate point is not important. To address this misconception, students might plot points with the coordinates switched. For example, referring to the graph from the previous question (Example 2) students might locate points (4, 6) and (6, 4) and then discuss the order they used to locate the points and how the order might change the amount of earnings on the chart. Provide opportunities for students to realize the importance of direction and distance, such as a student creating directions for others to follow as they plot points.

Geometry 5.G

Classify two-dimensional figures into categories based on their properties.

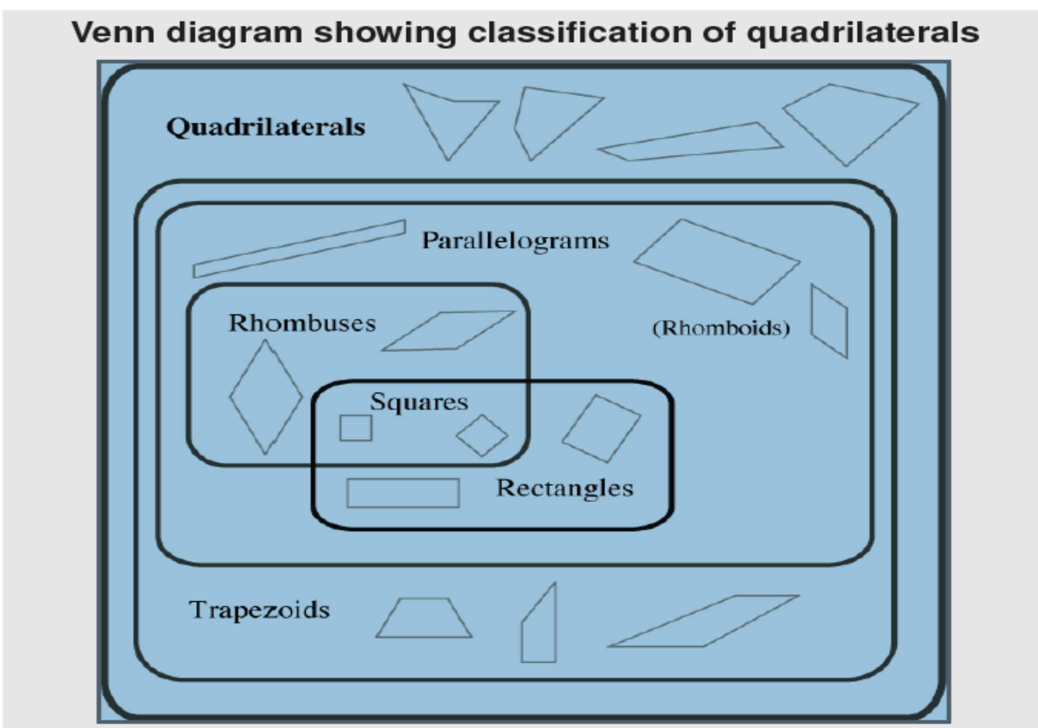
3. Understand that attributes belonging to a category of two-dimensional figures also belong to

all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

4. Classify two-dimensional figures in a hierarchy based on properties.

In prior years, students described and compared properties of two-dimensional shapes and built, drew, and analyzed these shapes. In grade five students broaden their understanding to reason about the attributes (properties) of two-dimensional shapes and to classify these shapes in a hierarchy based on properties (**5.G.4**). Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). For example, students conclude that all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides. In this way, they relate certain categories of shapes as subclasses of other categories (**5.G.3**).

For example:



(Progressions K-6 G 2012 and KATM 5th FlipBook 2012)

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, procedural skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way. Multiplication and division of whole numbers and fractions are an instructional focus in grades three through five.

To be prepared for grade six mathematics students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade four and have met the fluency expectations for the grade. For fifth graders, the expected fluency is to multiply multi-digit whole numbers with up to four-digits using the standard algorithm (**5.NBT.5**). These fluencies and the conceptual understandings that support them are foundational for work in later grades.

Of particular importance at grade five are concepts, skills, and understandings needed to understand the place value system (**5.NBT.1-4▲**); perform operations with multi-digit whole numbers and with decimals to hundredths **(5.NBT.5-7▲**); use equivalent fractions as a strategy to add and subtract fractions (**5.NF.1-2▲**); apply and extend previous understandings of multiplication and division to multiply and divide fractions. (**5.NF.3-7▲**); geometric measurement: understand concepts of volume and relate volume to multiplication and to addition (**5.MD.3-5▲**). In addition graphing points on the coordinate plane to solve real-world and mathematical problems (**5.G.1-2**) is an important part of a students' progress to algebra.

Fractions

Student proficiency with fractions is essential to success in algebra at later grades. By the end of grade five students should be able to add, subtract and multiply any two fractions and understand how to divide fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

Students should understand fraction equivalence and use their skills to generate equivalent fractions as a strategy to add and subtract fractions, with unlike denominators including mixed fractions. Students should use these skills to solve related word problems. This understanding brings together the threads of fraction equivalence (grades three through five) and addition and subtraction (kindergarten through grade four) to fully extend addition and subtraction to fractions.

By the end of grade five students know how to multiply a fraction or whole number by a fraction. Based on their understanding of the relationship between fractions and division, students divide any whole number by any nonzero whole number and express the answer in the form of a fraction or mixed number. Work with multiplying fractions extends from students' understanding of the operation of multiplication. For example, to multiply a/b x q (where q is a whole number or a fraction), students can interpret $\frac{a}{b}$ x q as meaning *a* parts of a partition of *q* into *b* equal parts. This interpretation of the product leads to a product that is less than, equal to or greater than *q*, depending on whether a/b<1, $\frac{a}{b}$ = 1 or $\frac{a}{b}$>1, respectively. For a/b<1, this result of multiplying contradicts earlier student experience with whole numbers, so this result needs to be discussed, explained, and emphasized.

Grade five students divide a unit fraction by a whole number or a whole number by a unit fraction. By the end of grade five students should know how to multiply fractions to be prepared for division of a fraction by a fraction in grade six.

Decimals

In grade five students will integrate decimal fractions more fully into the place value system as they learn to read, write, compare, and round decimals. By thinking about decimals as sums of multiples of base-ten units, students extend algorithms for multi-digit operations to decimals. By the end of grade five students understand operations with decimals to hundredths. Students should understand how to add, subtract, multiply, and divide decimals to hundredths using models, drawings, and various methods including methods that extend from whole numbers and are explained by place value meanings. The extension of the place value system from whole numbers to decimals is a major accomplishment for a student that involves both understanding and skill with base-ten units and fractions. Skill and understanding with adding, subtracting, multiplying, and dividing multi-digit decimals will culminate in fluency with the standard algorithm in grade six.

Fluency with Whole Number Operations

In grade five the fluency expectation is to multiply multi-digit whole numbers (one-digit numbers times a number with up to four-digits and two-digit numbers times two-digit numbers) using the standard algorithm. Students also extend their grade four work in finding whole-number quotients and remainders to the case of two-digit divisors. Skill and understanding of division with multi-digit whole numbers will culminate in fluency with the standard algorithm in grade six.

Volume

Students in grade five work with volume as an attribute of a solid figure and as a measurement quantity. Students also relate volume to multiplication and addition.

Students' understanding and skill with this work supports a learning progression leading to valuable skills in geometric measurement in middle school.

Grade 5 Overview

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 5

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

2.1 Express a whole number in the range 2-50 as a product of its prime factors. For example, find the prime factors of 24 and express 24 as 2x2x2x3. CA

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Number and Operations in Base Ten 5.NBT

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number and Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*
2. Solve word problems involving addition and subtraction of fractions referring to the same whole,

including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions, or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
    a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
    b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5. Interpret multiplication as scaling (resizing), by:
    a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
    b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n\ b)$ to the effect of multiplying a/b by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]
    a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
    b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.
    c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
    a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
    b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
    a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
    b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
    c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
4. Classify two-dimensional figures in a hierarchy based on properties.

The *Mathematics Framework* was adopted by the California State Board of Education on November 6, 2013. *The Mathematics Framework* has not been edited for publication.